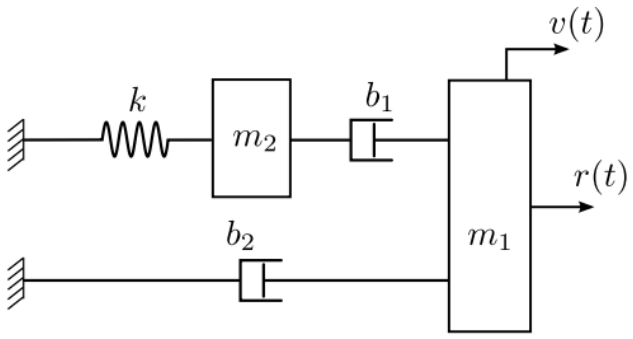
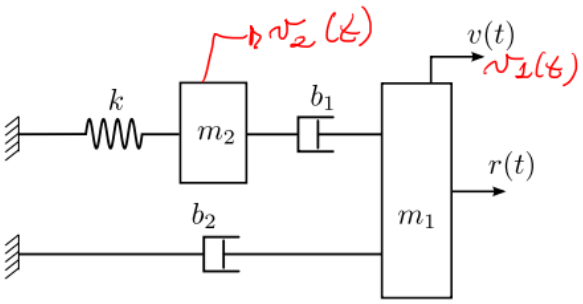
Homework 3 (solution)

1.

Find the transfer function $H(s) = \frac{V(s)}{R(s)}$ between the force $r(t)$ and the velocity of mass m_1.



1.1



Mass 1

$$r(t) - b_2 v_1 - b_1 (v_1 - v_2) = m \dot{v}_1$$

$$R(s) - b_2 V_1(s) - b_1 (V_1(s) - V_2(s)) = m s V_1(s)$$

$$\left[m_1 s + b_1 + b_2 \right] V_1(s) - b_1 V_2(s) = R(s) \quad (1)$$

Mass 2

$$b_1(v_1 - v_2) - b_1\int v_2\,dt = m_2\frac{dv_2}{dt}$$

$$m_2 s\,V_2(s) + b_1\left(V_2(s) - V_1(s)\right) + \frac{K}{s}V_2(s) = 0$$

$$-b_1 V_1(s) + \left(m_2 s + b_1 + \frac{K}{s}\right)V_2(s) = 0 \quad (2)$$

(1) $[m_1 s + (b_1 + b_2)]V_1(s) - b_1 V_2(s) = R(s)$

(2) $-b_1 V_1(s) + \left(m_2 s + b_1 + \frac{k}{s}\right)V_2(s) = 0$

$$\rightarrow\ V_2(s) = \frac{b_1}{m_2 s + b_1 + \frac{K}{s}}\,V_1(s) \quad (3)$$

(3) in (1) gives

$$[m_1 s + (b_1 + b_2)]V_1(s) - b_1\left(\frac{b_1}{m_2 s + b_1 + K/s}\right)V_1(s) = R(s)$$

solving.

$$\frac{V_1(s)}{R(s)} = \frac{m_2 s^2 + b_1 s + K}{(m_1 s + b_1 + b_2)(m_2 s^2 + b_1 s + K) - b_1^2 s}$$

2.

Calculate the natural frequency and damping ratio of the following transfer function

$$T(s) = \frac{1.05 \times 10^7}{2s^2 + 1.6 \times 10^3 s + 1.05 \times 10^7}$$

Determine:

→ Write the transfer function in standard form

→ Find the steady-state value for a step input

→ Calculate the natural frequency and damping ratio

2.1 Write the transfer function in standard form!!!

$$T(s) = \frac{0.525 \times 10^7}{s^2 + 0.8 \times 10^3 + 0.525 \times 10^7}$$

2.2 Find steady state value for a step input

for on step input

$$s\,T(s)\Big|_{s \to 0}$$

$$s \frac{1.05 \times 10^7}{2s^2 + 1.6 \cdot 10^3 s + 1.05 \times 10^7} \cdot \frac{1}{s}\Bigg|_{s \to 0}$$

$$= 1 //$$

2.3 find natural frequency (ω_n) and damping ratio (ζ)

$$\omega_n^2 = 0.525 \times 10^7 , \quad \boxed{\omega_n = 2291 \text{ rad/s}}$$

$$2\zeta\omega_n = 0.8 \cdot 10^3$$

$$\zeta = \frac{0.8 \cdot 10^3}{2 \times 2291} \rightarrow \boxed{\zeta = 0.17}$$